Analysis 1, Summer 2023

List 5

Increasing, decreasing, critical points, absolute extremes

113. (a) For what value(s) of x does $x^3 - 18x^2 = 0$?

 (b) For what value(s) of x does $3x^2 - 36x = 0$?

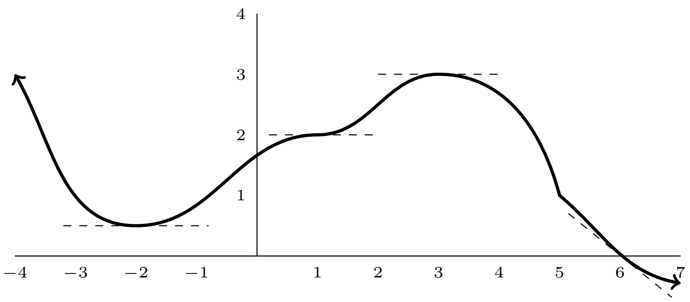
 (c) For what value(s) of x does $6x - 36 = 0$?

> A number c is a **critical point** of $f(x)$ if either $f'(c)$ does not exist or $f'(c) = 0$.
>
> If $f'(a) > 0$ then f is **increasing** at $x = a$.
> If $f'(a) < 0$ then f is **decreasing** at $x = a$.

114. What are the critical points of $x^3 - 18x^2$?

115. Find all the critical points of $8x^5 - 57x^4 - 24x^3 + 9$.

116. List all the critical points of the function graphed below (portions of its tangent lines at $x = -2$, $x = 1$, $x = 3$, and $x = 6$ are shown as dashed lines).



117. Is the function

$$f(x) = x^8 - 6x^3 + 29x - 12$$

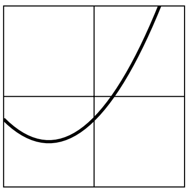
increasing, decreasing, or neither when $x = -1$?

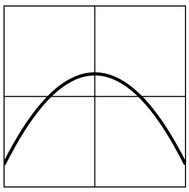
118. (a) On what (possibly infinite) interval or intervals is $2x^3 - 3x^2 - 12x$ decreasing?

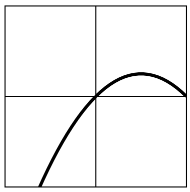
 (b) On what (possibly infinite) interval or intervals is $2x^3 - 3x^2 - 12x$ increasing?

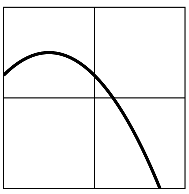
119. List all critical points of $f(x) = \frac{3}{4}x^4 - 7x^3 + 15x^2$ in the interval $[-3, 3]$.

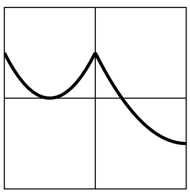
120. For each graph below, is there a critical point at $x = 0$?

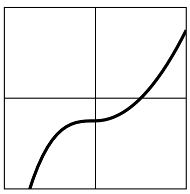
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

121. The derivative of

$$f(x) = \frac{4x+1}{3x^2-12} \qquad \text{is} \qquad f'(x) = \frac{-4x^2-2x-16}{3x^4-24x^2+48}.$$

Using this, find all the critical points of $f(x)$.

The derivative of $\sin(x)$ is $\cos(x)$. The derivative of $\cos(x)$ is $-\sin(x)$. In symbols, $\frac{\mathrm{d}}{\mathrm{d}x}\big[\sin(x)\big] = \cos(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\big[\cos(x)\big] = -\sin(x)$.

122. Give the derivative of $5\sin(x) + \frac{2}{3}\cos(x) - x^3 + 9$.

123. Give an equation for the tangent line to $y = \sin(x)$ at $x = \frac{\pi}{3}$.

124. Find all the critical points of

(a) $f(x) = x^2 - \cos(x)$.

(b) $f(x) = x + 2\cos(x)$.

(c) $f(x) = 2x + \cos(x)$.

(d) $f(x) = x^2 + x - \sin(x)$.

☆(e) $f(x) = x^2 + x + \cos(x)$.

To find the absolute extremes of a fn. on a closed, bounded interval: ① Find the critical points of f but *ignore critical points outside the interval.* ② Compute the value of f at the critical points *and* the endpoints of the interval. ③ The point(s) from ② with the largest f-value are absolute max, and point(s) with the smallest (i.e., most negative) f-value are absolute min.

125. On the interval $[-6, 3]$, find the absolute extremes of

$$2x^3 - 21x^2 + 60x - 20.$$

126. Find the absolute extremes of

$$x^4 - 4x^3 + 4x^2 - 14$$

on the interval $[-3, 3]$.

127. Find the absolute extremes of $x + 2\cos(x)$ with $0 \le x \le 2\pi$.

128. Find the absolute minimum and absolute maximum of

$$f(x) = \tfrac{3}{4}x^4 - 7x^3 + 15x^2$$

with $|x| \le 3$.

129. (a) Does the function $\dfrac{x-5}{x+2}$ have an absolute maximum on the interval $[-8, 4]$?

(b) Does the function $\dfrac{x-5}{\cos(x)+2}$ have an absolute maximum on $[-8, 4]$?

130. Give the derivative of each of the following:

   (a) $\frac{1}{2}x^4 + 4\sin(x)$
   (b) $2x^2 + 4\cos(x)$
   (c) $4x - 4\sin(x)$
   (d) $4 - 4\cos(x)$
   (e) $4\sin(x)$

131. A car drives in a straight line for 10 hours with its position after t hours being $24t^2 - 2t^3$ kilometers from its initial position. How far away is the farthest point the car reaches in 10 hours, and when does this occur?

Product Rule: $(fg)' = fg' + f'g$, also written $\frac{\mathrm{d}}{\mathrm{d}x}[fg] = f\frac{\mathrm{d}g}{\mathrm{d}x} + \frac{\mathrm{d}f}{\mathrm{d}x}g$.

132. For each function below, state whether its derivative can be found using *only* algebra, the Power Rule, the Constant Multiple Rule, and the Sum Rule. If so, give its derivative.

   (a) $4x^2 - 27x$
   (b) $4x^2 - 27$
   (c) $\sqrt{16x}$
   (d) $(x + \sqrt{7})^2$
   (e) 2^{x+7}
   (f) $\dfrac{5}{x}$
   (g) $\dfrac{3x}{6x + 15}$
   (h) $\dfrac{6x + 15}{3x}$

133. Using the Product Rule, give the derivative of $5^x \cdot \sin(x)$.

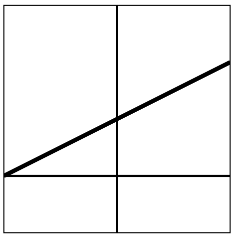
134. Use the Product Rule (twice) to find the derivative of $x^6 \cdot \cos(x) \cdot 2^x$.

135. True or false?

   (a) $(f + g)' = f' + g'$
   (b) $(f \cdot g)' = f' \cdot g'$
   (c) $(f \cdot g)' = f'g + fg'$
   (d) $\frac{\mathrm{d}}{\mathrm{d}x}(fg) = f\frac{\mathrm{d}g}{\mathrm{d}x} + g\frac{\mathrm{d}f}{\mathrm{d}x}$
   (e) $(f \cdot g)' = g'f' + gf'$
   (f) $(f/g)' = gf' - fg'$

136. Match the functions (a)-(f) to their derivatives (I)-(VI).

(a)



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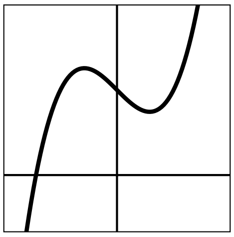

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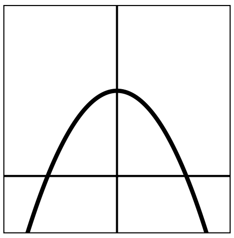

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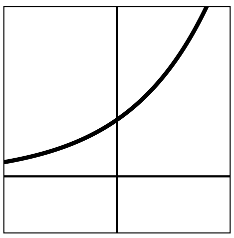

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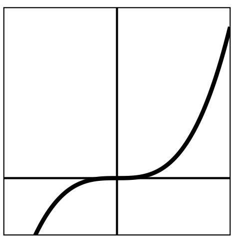

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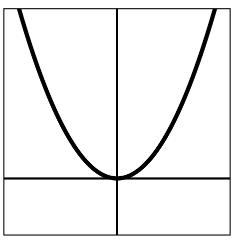
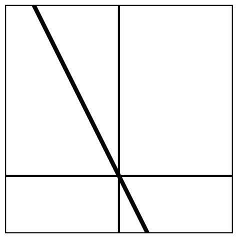


(I)



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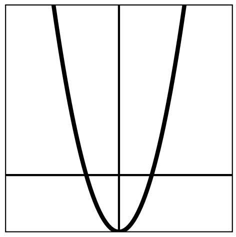

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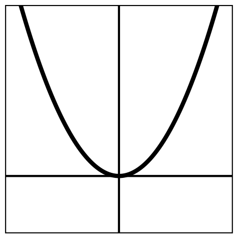

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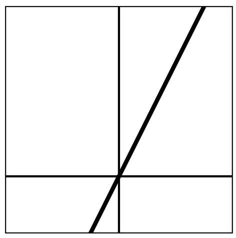

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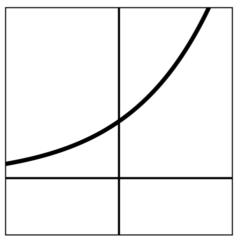

(VI)